MODEL REFERENCE ADAPTIVE CONTROL OF A ONE LINK FLEXIBLE ARM

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Abstract

Based on a model reference adaptive control approach, a robust controller for a one link flexible arm moving along a pre-defined trajectory is proposed. In order to satisfy the perfect model following conditions, the model is chosen from the linearized model of the system as optimally controlled. The nominal trajectory is commanded to the system by means of a dynamic filter. Simulation results for the prototype in the laboratory show the improvements obtained with the outer adaptive feedback loop with respect to a pure optimal control regulator. Robustness is finally tested by varying the nominal payload mass.

Introduction

Lightweight arms appear to be a challenging research topic to investigate in order to improve today's robot performance. Control is one key to efficient use of lighter arms, but it is limited by uncertainties in the arm's behavior and in the environment. In fact the main problem with lightweight structures is the flexible vibrations which are naturally excited as the arm is commanded to move.

The first step in designing a control system consists in developing a dynamical model for the flexible arm. A general dynamic modeling technique has been established in [1], based on a recursive Lagrangian-assumed modes method. If one is interested in the regulator control problem, that is to require that the arm reaches a pre-specified nominal state with satisfactory response performance, the approach of linearizing the dynamic equations, by assuming small motions around the nominal state and neglecting terms of higher order than one, proves effective, see [2] for instance.

On the other hand if one is concerned with controlling the arm while it is moving along a predefined path with given velocity and acceleration as regards the joint variables, the technique of linearizing the system is candidate to fail. Furthermore the attempt of linearizing around a sequence of nominal states seems too expensive from the computational standpoint, as well as not very robust as regards the overall nonlinear dynamics.

This paper describes a first research effort to control a one link flexible arm moving along a pre-defined trajectory. The approach adopted is essentially based on Model Reference Adaptive Control (MRAC) [3]. In order to assure the satisfaction of the so-called perfect model following conditions, the reference model is artificially chosen from the linearized system (2nd order terms neglected) as optimally controlled. Integral type adaptive actions assure the stability of the overall system, as it is proved via the Lyapunov direct method. However, since the reference model turns out not to be decoupled, the reference trajectory is forced into the system by means of a dynamic filter.

A case study with reference to the prototype existing in the laboratory, whose dynamic model is described in [4], shows that the control performs well with a fast trajectory to track. The whole nonlinear system is considered for simulation purpose. Moreover the control proves robust also to parameter variations, such as payload changes.

Last but not least it must be mentioned that full state availability is assumed for control synthesis. As a matter of fact the flexible time state variables can be obtained from strain gage measurements [5], whereas their derivatives need to be reconstructed by means of an observer [2].

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Problem Formulation

Nonlinear equations of motion for a flexible arm can be derived using the Lagrangian approach [1]. A solution of the flexible motion is assumed to be a linear combination of admissible functions multiplied by time dependent generalized coordinates. The flexible motion of a link is then described by

$$u(\eta,t) = \sum_{i=1}^{n} \phi_i(\eta)\delta_i(t), \tag{1}$$

where $\phi_i(\eta)$ are assumed to be the eigenfunctions of a clamped-free beam, $\delta_i(t)$ are the generalized coordinates, and η is any point along the undeformed link, see fig 1. Furthermore, assuming that the amplitudes of the higher modes of the flexible link are very small as compared to the first one, n = 2 will be accurate enough to describe the flexible motion.

The derivation of the dynamic equations for the one link arm follows then as in [4], i.e. (dropping the time dependence)

$$M(z)\ddot{z} = f(z,\dot{z}) + \tau, \tag{2}$$

where z is the vector of generalized coordinates (θ, δ_1, δ_2), M is the inertia matrix, f is the vector containing nonlinear dynamic terms (interactions of angular rates and deflections), and τ is the net input torque. Notice that in the model no actuator dynamics is considered, and no friction at joints nor in the structural vibrations are explicitly included. Define the full state vector

$$x^T = (z^T \mid \dot{z}^T), \tag{3}$$

and split the vector f in (2) as

$$f(x) = K(x)z + C(x)\dot{z}, \tag{4}$$

where K is an effective spring matrix, and similarly C is an effective damping matrix. The dynamical model of the flexible arm of fig. 1 can be expressed in state variable form as

$$\dot{x} = A(x)x + b(x)u \tag{5a}$$

$$A(x) = \left[\begin{array}{c|c} 0 & I \\ \hline M^{-1}K & M^{-1}C \end{array}\right] \quad b(x) = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline M^{-1}\begin{bmatrix}1\\0\\0\end{bmatrix} \end{array}\right]. \tag{5b}$$

At this extent it becomes clear why the tracking control problem is a hard one. In fact if the goal is just to require that the arm reaches a pre-specified nominal state, linearizing (5) around the nominal state leads naturally to an optimal control regulator, in which one can eventually specify the closed loop poles of the linearized system with an arbitrary degree of stability, see [2] for details. However, if it is desired to control the arm while it moves along a pre-defined trajectory, in terms of joint angle rates and accelerations, a different approach must be sought, rather than trying to linearize (5) around a sequence of nominal states.

In order to obtain good trajectory tracking and steady-state accuracy, a model reference adaptive control approach [3] is pursued in the following. The basic idea with this approach is to define a linear time-invariant reference model and directly synthesize a controller which assures that the error between the states of the system and those of the model tends to zero. To this purpose let

$$\dot{x}_m = A_m x_m + b_m u_m \tag{6a}$$

$$A_m \equiv \left[\begin{array}{c|c} 0 & I \\ \hline A_1 & A_2 \end{array}\right] \quad b_m = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline b_0 \end{array}\right] \tag{6b}$$

be a linear time-invariant reference model of the same dimension of the system described by eqs. (5).

Similarly to the work done on MRAC for rigid manipulators [6], it would seem appropriate to select a decoupled model in (6), i.e. $A_1 = -\mathrm{diag}(a_{11}\ a_{12}\ a_{13})$, $a_{1i} > 0$, $A_2 = -\mathrm{diag}(a_{21}\ a_{22}\ a_{23})$, $a_{2i} >$ 0. However the perfect model following conditions which are at the basis of a MRAC approach [3] cannot be satisfied independently from the particular values of A, A_m, b, b_m. The reason can be identified in a straightforward manner by observing that the system described in (5) does not have as many control inputs as nontrivial state variables (θ, δ_1, δ_2). In other words the lower block of vector b in (5b) is not a square block (a scalar in this case), and this causes the main problem in satisfying the perfect model following conditions, which in terms of the structure of the system are

$$(I - bb^+)(A_m - A) = 0$$

$$(I - bb^+)b_m = 0, \text{ for any } x,t, \tag{7}$$

where b^+ denotes the Penrose pseudo-inverse.

In the particular case of the system in (5), however, the nonlinear terms do not play a dominant role, thus it appears adequate to choose a reference model on the basis of the linearized model of the system (2nd order terms neglected) as optimally controlled; this approach will be outlined in the next section.

Control Law Development

Following the basic MRAC scheme in [3], a control for the overall system (5)- (6) is proposed in the form

$$u = u_1 + u_2 \tag{8a}$$

$$u_1 = -k_x^T x + k_u u_m \quad u_2 = -\Delta k_x^T x + \Delta k_u u_m, \tag{8b}$$

where u_1 is a linear model following control and u_2 represents the outer adaptive control which is devoted to assure the stability of the whole system. Under the action of control (8) the system (5) becomes

$$\dot{x} = A_s(x)x + b_s(x)u_m \tag{9a}$$

$$A_s = A - b(k_x^T + \Delta k_x^T) \quad b_s = b(k_u + \Delta k_u). \tag{9b}$$

Let then

$$e = x_m - x \tag{10}$$

be the error between model and system states; on reduction of (6) and (9), error dynamics is found to be

$$\dot{e} = A_m e + (A_m - A_s)x + (b_m - b_s)u \tag{11}$$

In order to satisfy the perfect model following conditions, the following should hold:

$$A_m = A - bk_x^T \quad b_m = bk_u. \tag{12}$$

If k_x^T is designed by means of optimal control for the linearized system in (5), obtained by neglecting the 2nd order nonlinear terms, and k_u is chosen equal to 1 for simplicity, the model (A_m,b_m) can be artificially selected so as to satisfy (12), with the intent to have a stable linear time-invariant model for the system, even if not a decoupled one. Supposing then that (12) holds, (11) becomes

$$\dot{e} = A_m e + b\Delta k\ x_x^T - b\Delta k_u u_m. \tag{13}$$

In order to guarantee the stability of the overall system, a candidate Lyapunov function is

$$V = e^T Pe + tr[(A_m - A_s)^T F_a^{-1}(A_m - A_s)] +$$

$$tr[(b_m - b_s)^T F_b^{-1}(b_m - b_s)], \tag{14}$$

where P, F_a, F_b are positive definite matrices. The derivative of V results then, accounting for (13),

$$\dot{V} = e^T(A_m^T P + PA_m)e +$$

$$2tr[(b\Delta k_x^T)^T(Pex^T - F_a^{-1}\dot{A}_s)] +$$

$$2tr[(-b\Delta k_u)^T(Peu_m - F_b^{-1}\dot{B}_s)]. \tag{15}$$

Setting as usual

$$A_m^T P + PA_m m = -H, \tag{16}$$

where H is a positive definite matrix, and assuming that $\Delta\dot{k}_x$, $\Delta\dot{k}_u >> \dot{A}, \dot{B}$ lead to

$$\dot{V} = b - e^T He + 2tr[(b\Delta k_x^T)^T(Pex^T + F_a^{-1} b\Delta\dot{k}_x^T)] +$$

$$2tr[(-b\Delta k_u)^T(Peu_m - F_b^{-1} b\Delta\dot{k}_u)]. \tag{17}$$

At this point the choice

$$\Delta\dot{k}_x^T = -(b^T F_a^{-1} b)^{-1} b^T Pex^T,$$

$$\Delta k_x^T(0) = \Delta k_{x0}^T \tag{18a}$$

$$\Delta\dot{k}_u = (b^T F_b^{-1} b)^{-1} b^T Peu_m,$$

$$\Delta k_u(0) = \Delta k_{u0} \tag{18b}$$

is easily proved to cancel the last two terms in (17), and assure that V be negative definite, thus guaranteeing that $e \to 0(x \to x_m)$.

The only problem now remains how to force the system to track a desired trajectory. This point has been addressed in [7] but, even with an equal number of controls and output variables, only a sinusoidal reference trajectory could be commanded of the rigid body motion. An inverse model technique of the type proposed in [6] cannot be adopted since the model (6), satisfying (12), turns out not to be decoupled. However, if $(\hat{\theta}t),\dot{\hat{\theta}}t))$ is the desired trajectory, an acceptable trade off can be achieved if the input u_m to the model is synthesized as the output of a dynamic filter of the PD type, i.e.

$$u_m = k_p(\hat{\theta} - \theta_m) + k_v(\dot{\hat{\theta}} - \dot{\theta}_m). \tag{19}$$

Proper selection of the gains k_p and k_v of this dynamic system set in front of the overall system assures that the joint variables of the reference model reproduce the desired joint angles and angle rates. More specifically, since the spectrum of the eigenvalues associated with the flexible time variables is distant enough from the origin (rigid body motion), forcing the input command (19) will have a dynamic effect mainly on the joint variables. Then, according to the above stability analysis, $(\theta,\dot{\theta})$ will track $(\theta_m,\dot{\theta}_m)$, and then $(\hat{\theta},\dot{\hat{\theta}})$, but at the same time $\delta 1$ and $\delta 2$ will be stabilized in virtue of the choice (12). A block diagram of the whole system control is sketched in fig. 2.

The Case Study

In the following a case study is developed for the one link flexible arm existing in the laboratory, whose dynamic model is fully descibed in [4].

As far as the joint angle trajectory is concerned, the arm is required to move from $\hat{\theta}_i = 0°$ to $\hat{\theta}_f = 90°$ in 2 seconds, following a standard trapezoidal velocity profile with maximum velocity $\dot{\hat{\theta}} = 60°/s$.

An optimal regulator with a prescribed degree of stability is designed, whose performance index is

$$J = \int_0^\infty \exp(-\alpha t)(x^T Qx + ru^2)dt \tag{20}$$

with the design terms chosen as Q = diag(100 100 100 100 100 5000), r = 1, α = 2. The feedback constant vector has then resulted k_x^T = (65.27 -176.13 -2937.23 27.72 -7.50 -67.27). k_u has been set to 1. Δk_x^T and Δk_u have been chosen as in (18) with F_a = 2I, F_b = .005I. Also H = I in (16). The gains of the filter in (19) have been chosen respectively as k_p = .6, k_v = .6.

Two different sets of simulations have been carried out, one with the above design parameters, and another one just with the constant feedback gains k_x^T and k_u, without any outer adaptive control. In order to analyze the control performance the whole nonlinear model has been simulated for the system (5) in both cases. A sampling rate of .1 ms has been adopted. Furthermore the robustness of the system control to parameter variations has been tested by doubling the payload mass, without changing the controls, the adaptive one and the optimal one respectively.

Figs. 3 through 10 illustrate the results obtained. They are quite self- explanatory. It can be recognized that the adaptive control performs better than the simple optimal control, as it contributes to obtain better tracking accuracy, smoother time-varying mode amplitudes and control torques. If the payload is doubled, it can be seen that control performance still remains satisfactory.

Conclusions

A model reference adaptive control has been presented for a one link flexible arm. In order to comply with the perfect model following conditions, the model has been set up as result of the linearized model of the system as optimally controlled. Resulting the reference model not a decoupled one, the desired joint angle trajectory is commanded through a dynamic filter set in front of the overall system. Full state availability has been supposed for control synthesis.

A case study has been developed for a prototype in the laboratory. Simulation results have shown the advantage of using an outer adaptive feedback control with respect to the pure optimal control and the robustness of the system control to payload variations.

It must be emphasized, however, that in the light of a more general MRAC for multiple link flexible manipulators the results obtained in this paper appear only partially satisfactory. As a matter of fact, in case of more degrees of freedom, the nonlinear coupling terms in the joint variables (which are not present in the one link case) become dominant, particularly at high speed, and control performance is likely to be derated.

This point, along with the problem of state reconstruction, or eventually considering output feedback, constitute two challenging research issues to investigate to a greater extent.

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References

[1] W.J. Book, "Recursive Lagrangian dynamics of flexible manipulators," Robotics Research, vol. 3, no. 3, Fall 1984.

[2] V. Sangveraphunsiri, "The optimal control and design of a flexible manipulator arm," Ph. D. dissertation, School of Mech. Eng., Georgia Tech, May 1984.

[3] Y.D. Landau, Adaptive Control: The Model Reference Approach, Marcel Dekker, Inc., NY, NY, 1979.

[4] G.G. Hastings and W.J. Book, "Verification of a linear dynamic model for flexible robot manipulators," IEEE Conf. Robotics Automation, San Francisco, CA, April 1986.

[5] G.G. Hastings and W.J. Book, "Experiments in optimal control of a flexible arm," ACC, Boston, MA, June 1985.

[6] A. Balestrino, G. De Maria and L. Sciavicco, "An adaptive model following control for robotic manipulators," Trans. ASME J. Dyn. Syst. Meas. Contr., vol. 105, Sept. 1983.

[7] D.R. Meldrum and M.J. Balas, "Direct adaptive control of a remote manipulator arm, "ASME Winter Annual Meeting, Miami, FL, Nov. 1985.

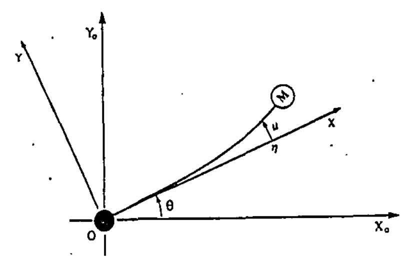

Fig. 1. The one link flexible arm.

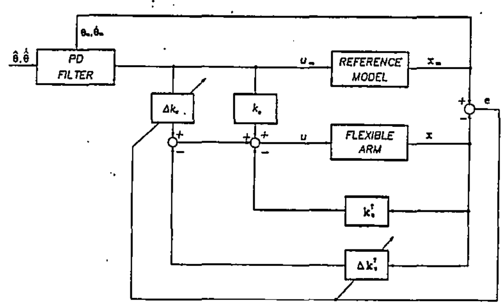

Fig. 2. Block diagram of the adaptive control scheme.

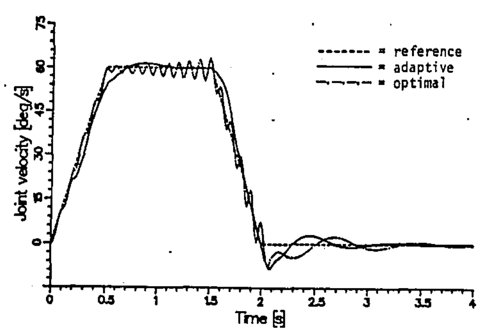

Fig. 3. Joint velocity profiles (nominal payload mass).

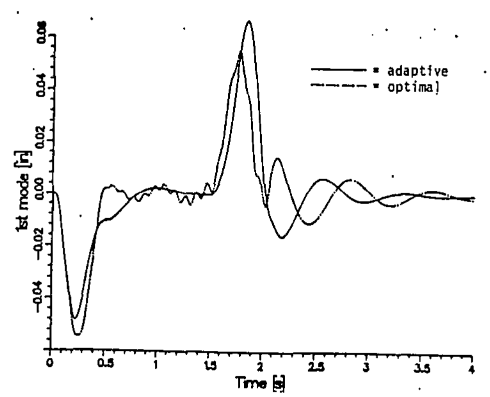

Fig. 4. First mode time amplitudes (nominal payload mass).

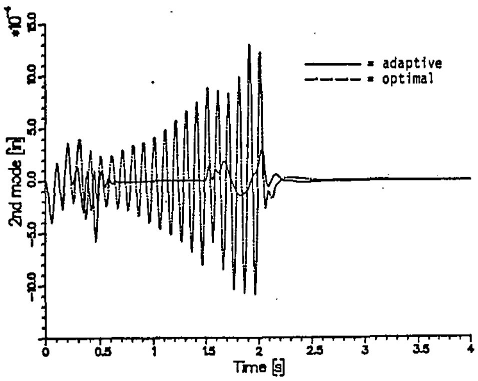

Fig. 5. Second mode time amplitudes (nominal payload mass).

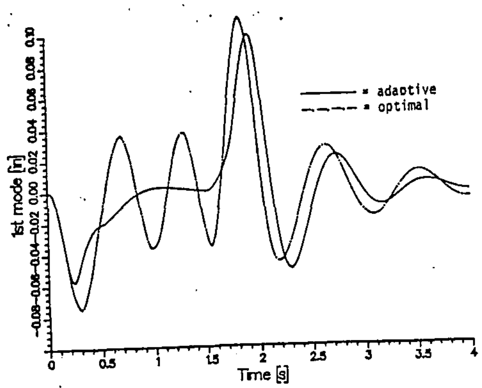

Fig. 8. First mode time amplitudes (doubled nominal payload mass).

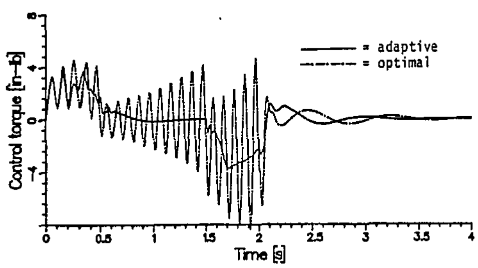

Fig. 6. Control torques (nominal payload mass).

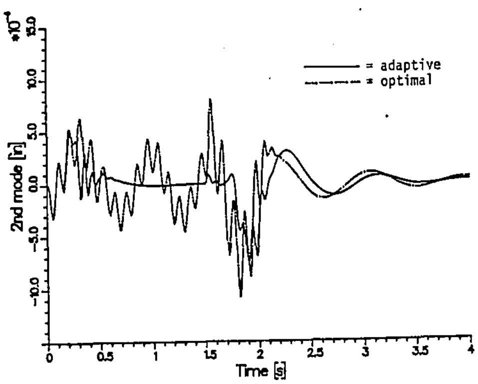

Fig. 9. Second mode time amplitudes (doubled nominal payload mass).

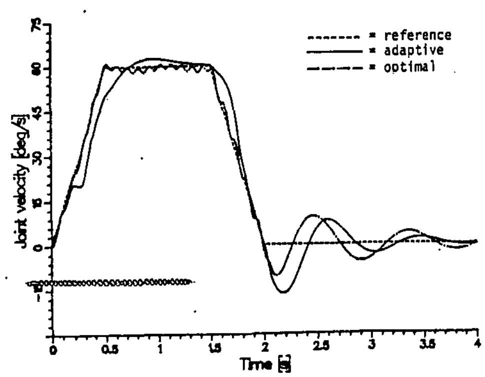

Fig. 7. Joint velocity profiles (doubled nominal payload mass).

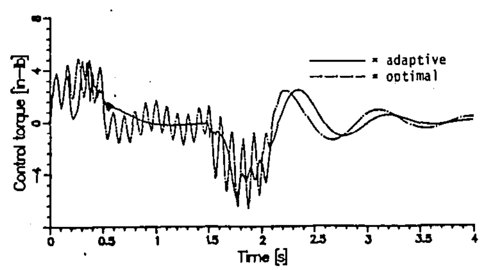

Fig. 10. Control torques (doubled nominal payload mass).